SUPERSPACE ACTIONS ON COADJOINT ORBITS OF GRADED INFINITE-DIMENSIONAL GROUPS

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The method of coadjoint orbits of infinite-dimensional Lie groups for derivation of geometric $D=2$ field theory actions is extended to the super-Kač–Moody and super-Virasoro groups in a manifestly (1,0) supersymmetric form. In this way we derive the explicit expressions for the actions of the supersymmetric chiral Wess–Zumino–Novikov–Witten (WZNW) model and of the induced supergravity (the super-gravitational Polyakov action). The latter action is also obtained in a different form by gauging the supersymmetric chiral SL(2, $\mathbb{R}$) WZNW model in the manifest superspace formalism.

1. Introduction

The recent papers by Faddeev and coworkers [1,2] and the closely related in spirit paper by Wiegmann [3] revived the interest in the method of coadjoint orbits of infinite-dimensional Lie groups for quantization of geometric field theory actions (see also refs. [4,5]).

In particular, it was shown in ref. [2] that using the standard Kirillov–Kostant symplectic structure [6] on the coadjoint orbits of the Virasoro ($D=2$ conformal) group one naturally obtains a geometric action which coincides with the Polyakov $D=2$ gravity action [7], i.e. the gravitational Wess–Zumino–Novikov–Witten (WZNW) action of the matter-induced $D=2$ gravity. Also, in ref. [2] a natural interpretation for the appearance of the SL(2, $\mathbb{R}$) symmetry in the Polyakov gravitational action was given by showing that the latter action may be obtained as a gauge-fixed field theory action from the ordinary SL(2, $\mathbb{R}$) WZNW model by gauging the lower-triangular subgroup B^- of SL(2, $\mathbb{R}$). This work was further extended in ref. [5] to the case of SL(n, $\mathbb{R}$) WZNW models and for $n \geqslant 3$ a relation was found with the W_n algebras of Zamolodchikov [8] [#1].

In a closely related development a very interesting reformulation of the usual method of coadjoint orbits was proposed by Wiegmann [3] by combining it with the method of generalized coherent states (see e.g. ref. [10]). In particular, by taking semidirect products of the current algebra based on the Poincaré group with the Virasoro group and their supersymmetric generalizations a new framework for quantization of (super)strings was provided.

In the present letter we propose a systematic manifestly (1,0) supersymmetric extension of the formalism of Alekseev and Shatashvili [2]. First, we consider the symplectic structure on the coadjoint orbits of the super-Kač–Moody group and obtain the $D=2$ supersymmetric chiral WZNW model in terms of (1,0) superfields. In component fields the latter model reduces to a sum of the ordinary bosonic chiral WZNW model plus decoupled

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[#1] Independently, in a recent paper [9] Polyakov provided a new deep insight into the geometric origin of these SL(n, $\mathbb{R}$) symmetries. His approach does not relay on the coadjoint orbit method but rather it is inspired by the ideas behind the spin↔isospin transmutation induced by magnetic monopoles.

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free chiral fermions in the adjoint representation of the underlying finite-dimensional group (an analogous decoupling occurs in the non-chiral (1,1) supersymmetric WZNW model [11]). Next, by gauging the supersymmetric chiral WZNW model for $G=SL(2,\mathbb{R})$ with respect to the Borel subgroup B^- of lower-triangular 2×2 matrices, we derive within a manifest superspace formalism a nonlinear superfield action whose purely bosonic limit exactly coincides with the gravitational Polyakov action [#2]. In the final section we find another extremely simple form of the supergravity Polyakov action by a systematic application of the coadjoint orbit method to the super-Virasoro (superconformal) group in a manifest superspace formalism.

2. Action on the coadjoint orbits of the super-Kač–Moody group

Let G be a finite-dimensional semi-simple Lie group and A its Lie algebra spanned by the generators $\{T_a\}$. The elements of the super-Kač–Moody algebra $\mathscr{A}$ with a central extension are pairs $(U(x,\theta), m)$ where the superfield $U(x,\theta)=U^a(x,\theta)T_a$ takes values in the underlying finite-dimensional algebra A and the parameter $x\in S^1$ [#3]. The number m denotes the component along the central extension. The elements of the dual space $\mathscr{A}^*$ are similarly pairs $(V(x,\theta), -k)$ with $V(x,\theta)=V^a(x,\theta)T_a$ and k a number. The pairing between $\mathscr{A}^*$ and $\mathscr{A}$ is defined as:

$$\langle (V,-k),(U,m)\rangle = \int \mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}(V(x,\theta)U(x,\theta))-km\,. \tag{1}$$

The component-field expansion of $U(x,\theta)$ and $V(x,\theta)$ is

$$U(x,\theta)=u_0(x)+\theta u_1(x)\,,\quad V(x,\theta)=v_1(x)+\theta v_0(x)\,, \tag{2}$$

where $u_0(x)$, $v_0(x)$ are bosonic, and $u_1(x)$, $v_1(x)$ are fermionic fields. The commutator between two elements of $\mathscr{A}$ is given by

$$[(U_1,m_1),(U_2,m_2)]=\left([U_1,U_2],\ -\frac{\mathrm{i}}{2\pi}\int \mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}[U_1(x,\theta)\mathrm{D}U_2(x,\theta)]\right), \tag{3}$$

where

$$\mathrm{D}=\frac{\partial}{\partial\theta}+\mathrm{i}\theta\frac{\partial}{\partial x} \tag{4}$$

is the usual super-covariant derivative and

$$[U_1(x,\theta),U_2(x,\theta)]=[U_1^aT_a,U_2^bT_b]=(f^c_{\ ab}U_1^aU_2^b)T_c \tag{5}$$

is the ordinary commutator in the finite-dimensional algebra A. The adjoint and the coadjoint actions of the super-Kač–Moody group are given, respectively, as

$$\mathrm{Ad}(G)(U,m)=\left(G(x,\theta)U(x,\theta)G^{-1}(x,\theta),\ m-\frac{\mathrm{i}}{2\pi}\int \mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}[U(x,\theta)G^{-1}(x,\theta)\mathrm{D}G(x,\theta)]\right) \tag{6}$$

$$\mathrm{Ad}^*(G)(V,-k)=\left(G(x,\theta)V(x,\theta)G^{-1}(x,\theta)-\mathrm{i}\frac{k}{2\pi}\mathrm{D}G(x,\theta)G^{-1}(x,\theta),\ -k\right) \tag{7}$$

[#2] In a recent paper [12], Bershadsky and Ooguri derived a supersymmetric generalization of the gravitational Polyakov action within the component-field formalism by gauging the OSp(1,2) WZNW model. Their result has a different form from ours, therefore, their action must be related to the present ones (eqs. (27) and (40)) by (nonlinear) field transformations.

[#3] Throughout this letter we shall follow the notations of ref. [2]. For a rigorous mathematical treatment of ordinary Kač–Moody (loop-) groups, see ref. [13].

where $G(x,\theta)$ is a superfield taking values in the finite-dimensional semi-simple group G corresponding to the algebra A. In component fields:

$$G(x,\theta)=\exp[\theta\psi(x)]g(x)=[1+\theta\psi(x)]g(x)\,, \tag{8}$$

where the fermionic field $\psi(x)$ takes values in A.

Now, let us briefly recall the general construction of actions of dynamical systems on coadjoint orbits of a Lie-group G [1–3]. On each coadjoint orbit $\mathcal{C}_X$ of G there exists a canonical symplectic two-form (i.e. closed and nondegenerate) [6] which is given by

$$\Omega_X=\tfrac{1}{2}\langle X,[Y,Y]\rangle\,, \tag{9}$$

where X (an element of the dual A*) is a generic point on this orbit and Y is a one-form taking values in A, the Lie-algebra of G. Y is specified as solution to the following basic equation:

$$\mathrm{d}X=\mathrm{ad}^*(Y)\,X\,. \tag{10}$$

In (10) $\mathrm{ad}^*(Y)$ denotes the infinitesimal adjoint action of A on A*:

$$\langle \mathrm{ad}^*(Y)\,X,Z\rangle=-\langle X,[Y,Z]\rangle \tag{11}$$

for any $Z\in$ A.

The symplectic form Ω_X is closed, hence (locally) exact:

$$\Omega_X=\mathrm{d}\alpha\,. \tag{12}$$

Therefore, the simplest action, describing the dynamical system defined by Ω_X and having the orbit $\mathcal{C}_X$ as its phase space, takes the form:

$$S=\int\alpha\,, \tag{13}$$

where the integral is over a closed curve on the orbit $\mathcal{C}_X$.

Thus, finding the geometric action (13) amounts to the problem of solving eq. (10) for the A-valued one-form Y. This is achieved by parametrizing the elements X of the orbit $\mathcal{C}_{X_0}$ through the group variables $g\in$ G as

$$X\equiv X(g)=\mathrm{Ad}^*(g)\,X_0\,, \tag{14}$$

where X_0 is a fixed generic point of this orbit. With this parametrization eq. (10) takes now the following useful form:

$$\mathrm{d}(\mathrm{Ad}^*(g)\,X_0)=\mathrm{ad}^*(Y)(\mathrm{Ad}^*(g)\,X_0)\,. \tag{15}$$

Returning to the super-Kač–Moody case, let us now denote

$$Y\equiv(\mathcal{Y}(x,\theta),-m_Y)\,,\quad X_0\equiv(V_0(x,\theta),-k) \tag{16}$$

and substitute the explicit expressions (3), (7) into eq. (15). One gets (suppressing the dependence on (x,θ))

$$\mathrm{d}\left(GV_0G^{-1}-\mathrm{i}\frac{k}{2\pi}\mathrm{D}GG^{-1},-k\right)=\left(\left[\mathcal{Y},GV_0G^{-1}-\mathrm{i}\frac{k}{2\pi}\mathrm{D}GG^{-1}\right]-\mathrm{i}\frac{k}{2\pi}\mathrm{D}\mathcal{Y},0\right). \tag{17}$$

The solution to (17) is straightforwardly obtained:

$$\mathcal{Y}(x,\theta)=\mathrm{d}G(x,\theta)G^{-1}(x,\theta)\,,\quad m_Y=0 \tag{18}$$

and it is the natural supersymmetric generalization of the purely bosonic one-form in ref. [2].

Now, using (14), (7) and (18), the result for the symplectic form Ω (9) reads

$$\Omega=\mathrm{d}\left[\int \mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}\left(-V_0(G^{-1}\mathrm{d}G)+\mathrm{i}\frac{k}{4\pi}(G^{-1}\mathrm{d}G)(G^{-1}\mathrm{D}G)-\mathrm{i}\frac{k}{4\pi}d^{-1}(G^{-1}\mathrm{D}GG^{-1}\mathrm{d}G\wedge G^{-1}\mathrm{d}G)\right)\right]. \quad (19)$$

Hence, the corresponding action (13) takes the form

$$W'_{\mathrm{swznw}}=\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}\left(-V_0(G^{-1}\partial_t G)+\mathrm{i}\frac{k}{4\pi}(G^{-1}\partial_t G)(G^{-1}\mathrm{D}G)\right)$$
$$-\mathrm{i}\frac{k}{4\pi}\int_0^1 \mathrm{d}s\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}\{(G^{-1}\mathrm{D}G[G^{-1}\partial_t G, G^{-1}\partial_s G]\}. \quad (20)$$

Here t denotes the parameter of the one-dimensional curve of integration in (13), $G\equiv G(t,x,\theta)$ is a (1,0) $D=2$ group-valued superfield, and the second term on the RHS of (20) is the supersymmetric analogue of the well-known multivalued WZNW functional where

$G(s=0,t,x,\theta)=1$, $G(s=1,t,x,\theta)=G(t,x,\theta)$.

In component fields (8) the action (20) looks as (for the particular case of $V_0=0$)

$$W'_{\mathrm{swznw}}=W'^{\mathrm{chiral}}_{\mathrm{wznw}}[g]-\mathrm{i}\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{tr}(\psi\partial_t\psi), \quad (21)$$

where

$$W'^{\mathrm{chiral}}_{\mathrm{wznw}}[g]\equiv-\frac{k}{4\pi}\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{tr}(g^{-1}\partial_t g\,g^{-1}\partial_x g)+\frac{k}{4\pi}\int_0^1 \mathrm{d}s\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{tr}(g^{-1}\partial_x g[g^{-1}\partial_t g, g^{-1}\partial_s g]) \quad (22)$$

is the chiral bosonic WZNW action [14] (after the natural identification $t\equiv x^+$, $x\equiv x^-$ of t, x and $D=2$ light-cone coordinates). A similar result (decoupling of the superpartners of the group-valued field $g(t,x)$) was previously obtained in ref. [11] for the usual (non-chiral) (1,1) supersymmetric WZNW model.

3. Gauging of the SL(2, ℝ) supersymmetric chiral WZNW model

Following ref. [2] we now gauge the supersymmetric chiral WZNW model (20) (with $V_0=0$) for the group SL(2, ℝ) with respect to the B^-, the Borel subgroup of lower-triangular 2×2 matrices. This is achieved by imposing a gauge-symmetry constraint through a superfield Lagrange multiplier $\mathcal{A}(t,x,\theta)$ in the action (20).

$$W'_{\mathrm{gauged}}=W'^{\mathrm{chiral}}_{\mathrm{swznw}}[G]+\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}\theta\,\mathrm{tr}(\mathcal{A}(\partial_x GG^{-1}-\sigma^+)), \quad (23)$$

where

$$\mathcal{A}\equiv\begin{pmatrix}0 & 0\\ \mathcal{M}(x,\theta) & 0\end{pmatrix}, \quad \sigma^+\equiv\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}.$$

Let us note that $\mathcal{M}(x,\theta)$ is a fermionic superfield taking values in the algebra sl(2, ℝ). Analogously to the purely bosonic case [2], the action (23) possesses a B-*superfield* gauge symmetry:

$$G(t,x,\theta)\to H(t,x,\theta)G(t,x,\theta),$$

$$\mathcal{A}(t,x,\theta)\to H(\cdot)\mathcal{A}(\cdot)H^{-1}(\cdot)+\frac{k}{2\pi}\partial_t H(t,x,\theta)H^{-1}(t,x,\theta), \quad (24)$$

$$H(t,x,\theta)\equiv\begin{pmatrix}1 & 0\\ h(t,x,\theta) & 1\end{pmatrix}. \tag{24 cont'd}$$

Also, as in the purely bosonic case, there is an additional infinite-dimensional *global* [#4] symmetry

$$G(t,x,\theta)\to G(t,x,\theta)Q(t)\,, \tag{25}$$

where $Q(t)$ is a (x,θ)-independent element of SL(2, ℝ).

The SL(2, ℝ) group-valued (1,0) superfield $G(t,x,\theta)$ may be parametrized (in the connected component of the unit element) as

$$G(t,x,\theta)=\begin{pmatrix}1 & 0\\ \Omega(t,x,\theta) & 1\end{pmatrix}\begin{pmatrix}A(t,x,\theta) & 0\\ 0 & A^{-1}(t,x,\theta)\end{pmatrix}\begin{pmatrix}1 & \mathscr{F}(t,x,\theta)\\ 0 & 1\end{pmatrix} \tag{26}$$

where Ω, A, $\mathscr{F}$ are ordinary bosonic (1,0) superfields.

Imposing as in ref. [2] a gauge fixing condition

$$\Omega(t,x,\theta)=\tfrac{1}{2}\partial_x\left(\frac{1}{\mathscr{F}(t,x,\theta)}\right)$$

for the B$^-$-gauge symmetry (24) which is covariant under the global symmetry (25), and inserting the parametrization (26) into (23) (upon elimination of the Lagrange multiplier $H(t,x,\theta)$, one obtains

$$W_{\text{gauged}}=-\mathrm{i}\frac{k}{8\pi}\int \mathrm{d}t\,\mathrm{d}x\,\mathrm{d}\theta\,\frac{\partial_t\mathscr{F}}{\mathscr{F}'}\left[3\frac{\mathrm{D}\mathscr{F}''}{\mathscr{F}'}-8\frac{\mathrm{D}\mathscr{F}'\mathscr{F}''}{\mathscr{F}'^2}+2\frac{\mathrm{D}\mathscr{F}}{\mathscr{F}'}\left(3\frac{\mathscr{F}''^2}{\mathscr{F}'^2}-\frac{\mathscr{F}'''}{\mathscr{F}'}\right)\right]. \tag{27}$$

Here and below primes indicate differentiation with respect to x.

Inserting into (27) the component-field expansion $\mathscr{F}=F_0(t,x)+\theta F_1(t,x)$ we get

$$W_{\text{gauged}}=\frac{k}{8\pi}\int \mathrm{d}t\,\mathrm{d}x\,\frac{\partial_t F_0}{F_0'}\left(\frac{F_0'''}{F_0'}-2\frac{F_0''^2}{F_0'^2}\right)+(\text{terms containing } F_1)\,. \tag{28}$$

The first term on the RHS of (28) is immediately recognized as the gravitational Polyakov-WZNW action [7] (matter-induced $D=2$ gravity) modulo a nonlinear field redefinition [2]. Therefore, the action (27) provides the explicit manifest (1,0) superspace form of the induced $D=2$ supergravity [15,16].

4. Action on the coadjoint orbits of the super-Virasoro group

Now we shall derive an alternative form of the $D=2$ induced supergravity action (27) generalizing, as in section 2, the method of the coadjoint orbits of the usual Virasoro (conformal) group [2] to the super-Virasoro case [#5].

First, let us recall [18] that the action of the super-Virasoro group (i.e., the finite superconformal transformations) is given by the graded pair of superfields $\tilde{X}(x,\theta)$, $\tilde{\Theta}(x,\theta)$ (with $x\in S^1$):

$$z=(x,\theta)\to\tilde{X}=\tilde{X}(x,\theta)=\tilde{X}_0(x)+\theta\tilde{X}_1(x)\,,\quad \tilde{\Theta}=\tilde{\Theta}(x,\theta)=\tilde{\Theta}_1(x)+\theta\tilde{\Theta}_0(x)\,, \tag{29}$$

subject to the constraint

$$\mathrm{D}\tilde{X}-\mathrm{i}\tilde{\Theta}\mathrm{D}\tilde{\Theta}=0\,, \tag{30}$$

[#4] This is a global symmetry in a sense that its action does not trivialize itself on the boundary of $D=2$ space-time.

[#5] For a comprehensive description of orbits of the ordinary (bosonic) Virasoro group, see ref. [17] and references therein.

where D denotes the super-covariant derivative given in (4).

Henceforth we shall use the shorthand notation $\tilde{Z} \equiv (\tilde{X}, \tilde{\Theta})$. Eq. (30) implies the following relations for the component-fields:

$$\tilde{\Theta}_0(x) = (\tilde{X}_0')^{1/2}\left(1 - \mathrm{i}\frac{\tilde{X}_1\tilde{X}_1'}{\tilde{X}_0'^2}\right), \quad \tilde{\Theta}_1(x) = -\mathrm{i}\frac{\tilde{X}_1}{(\tilde{X}_0')^{1/2}}, \tag{31}$$

where the prime indicates as usually differentiation with respect to x.

The infinitesimal superconformal transformations form the infinite-dimensional graded Lie algebra with central extension – the super-Virasoro algebra $\mathscr{V}$. Its elements are pairs $(g(x, \theta), n)$, where $g(x, \theta)$ is a shorthand notation for the vector field $g(x, \theta)\partial_x - \frac{1}{2}\mathrm{i}Dg(x, \theta)D$ on the super-circle, parametrized by (x, θ) and the number n is the component along the central extension. The commutator of two elements of $\mathscr{V}$ is given by

$$[(g_1, n_1), (g_2, n_2)] = \left(g_1g_2' - g_1'g_2 - \tfrac{1}{2}\mathrm{i}Dg_1\,Dg_2,\ -\frac{\mathrm{i}}{12\pi}\int \mathrm{d}x\,\mathrm{d}\theta(Dg_1''g_2 - g_1Dg_2'')\right). \tag{32}$$

The elements of the dual space $\mathscr{V}^*$ are pairs $(B(x, \theta), c)$ with $B(x, \theta)$ being a fermionic superfield of conformal dimension $\frac{3}{2}$ and c is a number. The dual is defined with respect to the bilinear form

$$\langle (B, c), (g, n)\rangle = \int \mathrm{d}x\,\mathrm{d}\theta\, B(x, \theta)g(x, \theta) + cn. \tag{33}$$

The adjoint and the coadjoint actions of the super-Virasoro group are given, respectively, as

$$\mathrm{Ad}(\tilde{Z})(g, n) = \left(\frac{1}{(D\tilde{\Theta})^2}g(\tilde{X}(x, \theta), \tilde{\Theta}(x, \theta)),\ n + \frac{1}{3\pi}\int \mathrm{d}x\,\mathrm{d}\theta\, S(z; \tilde{Z}^{-1})g(x, \theta)\right), \tag{34}$$

$$\mathrm{Ad}^*(\tilde{Z})(B, c) = \left((D\theta)^3B(\tilde{X}(x, \theta), \tilde{\Theta}(x, \theta)) + \frac{c}{3\pi}S(z; \tilde{Z}), c\right). \tag{35}$$

In (34) and (35) $S(z; \tilde{Z})$ is the super-schwarzian derivative [18]

$$S(z, \tilde{Z}) \equiv S((x, \theta); (\tilde{X}, \tilde{\Theta})) = -\frac{\tilde{\Theta}''}{D\tilde{\Theta}} + 2\frac{(D\tilde{\Theta}')\tilde{\Theta}'}{(D\tilde{\Theta})^2}, \tag{36}$$

and $\tilde{Z}^{-1}$ in (34) indicates the inverse finite conformal transformation.

Now, we have to solve eq. (15) for the $\mathscr{V}$-valued one-form $Y \equiv (\mathscr{Y}(x, \theta), n_Y)$. In the present case the generic point on the coadjoint orbit X_0 is given by

$$X_0 \equiv (B_0(x, \theta), c). \tag{37}$$

Plugging in (15) the expressions (32)–(37) one gets $n_Y = 0$ plus a consistent overdetermined system of four equations for $\mathscr{Y}(x, \theta)$ (each term in front of $B_0(\tilde{Z})$, $\partial_xB_0(\tilde{Z})$, $DB_0(\tilde{Z})$ and in front of $c/3\pi$ should independently vanish). We easily find the following simple solution:

$$\mathscr{Y}(x, \theta) = \frac{\mathrm{d}\tilde{X} + \mathrm{i}\tilde{\Theta}\mathrm{d}\tilde{\Theta}}{(D\tilde{\Theta})^2}, \tag{38}$$

which yields for the symplectic form Ω (9):

$$\Omega = \mathrm{d}\left[\int \mathrm{d}x\,\mathrm{d}\theta\left(-B_0(\tilde{Z})D\tilde{\Theta}(\mathrm{d}\tilde{X} + \mathrm{i}\tilde{\Theta}\mathrm{d}\tilde{\Theta}) + \frac{c}{3\pi}\frac{\partial_xD\tilde{\Theta}}{(D\tilde{\Theta})^2}\mathrm{d}\tilde{\Theta}\right)\right]. \tag{39}$$

To get the result (39) we performed appropriate partial integrations and used the constraint (30).

Thus, the final form of the geometric action on the super-Virasoro coadjoint orbits (the supergravity Polyakov action) W_{sp} acquires the form

$$W'_{sp} = \int dt\, dx\, d\theta \left(-B_0(\tilde{X}, \tilde{\Theta}) D\tilde{\Theta}(\partial_t \tilde{X} + i\tilde{\Theta}\partial_t\tilde{\Theta}) + \frac{c}{3\pi} \frac{\partial_x D\tilde{\Theta}}{(D\tilde{\Theta})^2} \partial_t \tilde{\Theta} \right) . \tag{40}$$

For the particular case of taking the generic point on the orbit $B_0(\tilde{X}, \tilde{\Theta})=0$ and inserting into (40) the component-field expansions (29), (31) of $\tilde{X}(t, x, \theta)$ and $\tilde{\Theta}(t, x, \theta)$ we get

$$W'_{sp} = -\frac{c}{12\pi} \int dt\, dx \frac{\partial_t \tilde{X}_0}{\tilde{X}'_0} \left(\frac{\tilde{X}'''_0}{\tilde{X}'_0} - 2\frac{\tilde{X}''^2_0}{\tilde{X}'^2_0} \right) + (\text{terms containing } \tilde{X}_1) . \tag{41}$$

Thus, the purely bosonic limit of the super-Virasoro geometric action (the first term on the RHS of (41)) exactly coincides with the usual bosonic gravitational Polyakov action, as it was the case for the gauged SL(2, $\mathbb{R}$) supersymmetric chiral WZNW model (27), (28). Hence, the super-Virasoro geometric action (40) (for $B_0(\tilde{X}, \tilde{\Theta})=0$) on the coadjoint super-Virasoro orbits is precisely an explicit manifest superspace form of the matter-induced $D=2$ supergravity action.

Let us note that the action (40) describes the same physical theory as the action (27), although their functional forms are quite different. Indeed, (40) is written in terms of a fermionic (1,0) superfield $\tilde{\Theta}(t, x, \theta)$, whereas (27) is given in terms of a bosonic (1,0) superfield $\mathcal{F}(t, x, \theta)$. Presumably, there should exist a (highly nonlinear) superfield transformation bringing the form of the action (40) into the form (27).

To conclude , let us point out that, even before one attempts to quantize the above geometric actions, a lot of open questions still remain to be investigated. In particular, for the super-Virasoro group the explicit expressions of $B_0(\tilde{Z})$, which describe generic points on the coadjoint orbits, are still lacking for most classes of orbits [19]. (This situation is much the same as for the usual Virasoro group [17,2]). On the other hand, the knowledge of nontrivial $B_0(\tilde{Z})$ would yield superspace geometric actions with additional symmetries related to the stationary subgroups of $B_0(\tilde{Z})$. ALso, the generalization of the above manifest superspace formalism for arbitrary (p, q) $D=2$ supersymmetry will not be straightforward (and, presumably, will need introduction of auxiliary variables [20]).

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